

# **Exchange Rate Exposure and Optimal Hedging Strategies when Interest Rates are Stochastic: a Simulation-Based Approach**

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## **Abstract**

In this paper I analyze the problem faced by an investor expecting to receive a cash flow in a foreign currency. The investor is assumed to be exposed to long-term exchange rate risk, having no access to long-term forward contracts to hedge perfectly. Under non stochastic interest rates the investor is able to hedge perfectly using short-term forward contracts, but perfect hedging is not possible when we consider interest rates to be stochastic. I present here a simulation-based methodology to obtain optimal hedging under stochastic interest rates (i.e. when perfect hedging can not be reached). Then, I explore how the quality of the hedging to be reached depends on some key factors such as the volatility of the exchange rate, the volatility of the interest rates, and the degree of correlation among the stochastic variables considered.

## **Extracto**

En este artículo analizo el problema que enfrenta un inversionista que recibirá un flujo de caja en moneda extranjera en una cierta fecha futura. Este inversionista está expuesto entonces a riesgo cambiario, en particular si no hay disponibilidad de contratos

*forward* con extensión similar que le permitan hacer cobertura perfecta. El riesgo puede ser eliminado en un mundo con tasas de interés determinísticas incluso con contratos *forward* de corto plazo, pero la cobertura perfecta no es posible si hay tasas de interés estocásticas. Aquí se presenta una metodología que permite obtener en este segundo escenario la mejor cobertura posible, a la que se denomina cobertura óptima. Luego se evalúa para un ejemplo concreto la calidad de esta cobertura y cómo depende esta de ciertos parámetros clave tales como las volatilidades del tipo de cambio y de las tasas de interés, o el grado de correlación entre las variables consideradas.

## 1. Introduction

Firms who have their sales indexed to a foreign currency and their expenses indexed to a local currency are exposed to exchange rate risk. Those investors could protect themselves by taking short positions in exchange rate forward or futures contracts. Normally, the hedging would not improve the expected outcome in local currency, but would make the cash flows more certain and reduce exposition to risk.

Previous research on why firms hedge, such as the ones performed by Smith and Stultz (1985), Bessembinder (1991), Froot, Scharfstein and Stein (1993), and Mello and Parsons (1995) has identified the desires to minimization of the variance of future cash flows, the reduction of the volatility of taxable income, the desire to reduce dispersion of accounting earnings, or even the hope of being able to avoid financial distress as the main reasons for hedging. Neuberger (1999) assumes the desire for hedging comes from risk averse agents wishing to maximize expected utility.

The use of forward or futures contracts to hedge against exchange rate risk works less than perfectly in the real world for at least two reasons: First, the exchange rate we want to hedge from may not be the same as the exchange rate considered in the forward contract. In this case the quality of the hedge will critically depend on how closely correlated are those two exchange rates. Second, the date of expiration or maturity of the future or forward contracts available to perform the hedging may not coincide exactly with the particular date in the future we will receive the foreign

currency. This could happen for example if there are only short-term future or forward contracts available to hedge against long-term exchange rate exposure.

It has been shown that using short-term forward contracts to hedge against long-term exposure would allow reaching perfect hedging if interest rates are non-stochastic. For example Brennan and Crew (1995) present a model where a simple or tailed stack and roll strategy allows reaching perfect hedge when deterministic interest rates are assumed. Neuberger (1999) shows that even with deterministic interest rates, perfect hedging would not be reached unless forward prices can be predicted in advance and perfectly. He assumes that the price at which a contract first trades is a stochastic function of the prices of other contracts already trading in the market, and by assuming that the expected value of the opening price is a linear function of the prices of other contracts, the author proves that there is a unique hedging strategy independent of the agent's utility function, that dominates all the other strategies in the sense of second order stochastic dominance. The methodology proposed by Neuberger allows removing 85% of the risk of a six-year oil supply commitment. Schwartz (1997) compares three models of the stochastic behavior of commodity prices, taking account of mean reversion. He also analyzes the implications of those models for hedging risk exposure.

In a previous paper Castillo and Lefort (2002) show how a firm could obtain optimal hedging against exchange rate exposure using short-term forward contracts when it is known that a single amount of foreign currency will be received in a certain long term-future period  $T$ , and interest rates are stochastic. The hedging methodology proposed by Castillo and Lefort (2002) is constructed assuming that interest rates follow a Cox, Ingersoll, Ross process, and is implemented to show that by using a combination of 3 and 12 month forward contracts a Chilean company would be able to hedge 98% of the exchange rate faced when a cash flow in US\$ dollars is expected to be received 10 years from now .

In this paper I present a simulation-based methodology that can be applied to find the perfect hedging strategy when the exchange rate and the interest rates are all stochastic. The methodology assumes interest rates follow a Vasicek process, allowing us to use analytical expressions for the

value of the forward contracts. The quality of the hedging and the dependence of the quality of the hedging on some key parameters is also reviewed.

This paper is organized as follows. In section 2 the long term exchange rate risk problem faced by the investor is described, and the perfect hedging strategy using short term forward contracts is presented and shown to be reachable when interest rates are non stochastic. In section 3 our proposed methodology to find the optimal hedging strategy under a stochastic interest rates scenario is described. Section 4 reports the implementation of the optimal hedging strategy described in section 3 to a particular case and explores how good is the optimal hedging strategy under a series of different scenarios. Section 5 concludes the paper.

## **2. Exchange Rate Exposure and Hedging when Interest Rates are Non-Stochastic**

Let's suppose we have an investment that will generate a cash flows of one unit of a foreign currency  $T$  periods from now<sup>1</sup>, and let's assume that only one-period forward contracts are available. Hedging this long-term exchange rate risk perfectly using one-period forward contracts would be possible through the following procedure: The investor would have to take at each period  $t$ , starting now at  $t=0$ , and at each period  $t$  from  $t=1$  to  $t=T-1$ ,  $h_{t+1}$  positions in forward contracts. Those positions, taken in period  $t$ , will have to be rebalanced at the expiration of the forward contracts, in period  $t+1$ .

Considering the unit of exchange rate to be received in  $T$ , and assuming that the gains or losses generated by all the positions taken in one-period forwards contracts from  $t=0$  to  $t=T-1$  are transformed to a cash flow in period  $T$ , the following expression represents the total cash flow that the company will generate at  $T$ :

<sup>1</sup> We will assume we are currently at  $t=0$ .

$$CF_T = S_T + \sum_{t=0}^{T-1} {}_t h_{t+1} * (\bar{S}_{t+1} - {}_t F_{t+1}) * (1 + r_L)^{T-1-t} \quad (1)$$

where  $S_T$  represents the value of one unit of the underlying asset, in this case one unit of exchange rate, at time  $T$ ;  ${}_t F_{t+1}$  is the forward price fixed at  $t$  for a contract expiring at  $t+1$ ;  ${}_t h_{t+1}$  represents the number of positions taken in those forward contracts in period  $t$ ; and  $r_L$  corresponds to the local interest rate<sup>2</sup>. Under deterministic interest rates it is possible to assume that  ${}_t F_{t+1}$ , defined as the forward price fixed at  $t$  for a contract expiring at  $t+1$  will be computed as:

$${}_t F_{t+1} = S_t * \frac{(1 + r_L)}{(1 + r_f)} \quad (2)$$

where  $r_f$  corresponds to the foreign interest rate. Replacing (2) in (1) we obtain the cash flow in  $T$  as a function of the value of the underlying asset  $S$  in each period  $t=1$  to  $t=T$ . Solving the hedging problem now requires finding the  $h$  values that would make this cash flow free of the uncertainty faced at  $t=0$  regarding the values of  $S_t$  in periods  $t=1$  to  $T-1$ .

The solution to this optimization problem can be outlined in the expression for the hedging positions presented next:

$${}_t h_{t+1} = -\frac{1}{(1 + r_f)^{T-1-t}} \quad (3).$$

From here we conclude that by taking the short positions described by equation (3) in each period from  $t=0$  to  $t=T-1$  the cash flow in  $T$  would no longer depend on the values the underlying asset could take in the future. Equation (3) describes the perfect hedging strategy under deterministic interest rates. The next section describes the methodology I am proposing to find the optimal hedging strategy when interest rates follow a known stochastic process.

<sup>2</sup>The local interest rate is the risk free rate to lend or borrow in the local currency.

### 3. Exchange Rate Exposure and Hedging when Interest Rates are Stochastic

When interest rates are stochastic we are no longer able to reach perfect hedging for a long-term exchange rate exposure using short-term forward contracts. In this section we will show how we can develop an optimal hedging strategy under an stochastic interest rates scenario.

We will adapt the three-factor model developed by Schwartz (1997), assuming that the exchange rate follows a geometric Brownian process, and assuming that the interest rates follow mean-reverting processes as the one modeled by Vasicek (1977). The joint-stochastic-risk-adjusted process for the variables under the equivalent martingale measure can be expressed as:

$$dS = (r_L - r_F)S dt + \sigma_1 S dz_1 \quad (4)$$

$$dr_F = \kappa(\alpha - r_F) dt + \sigma_2 dz_2 \quad (5)$$

$$dr_L = \gamma(\beta - r_L) dt + \sigma_3 dz_3 \quad (6)$$

$$dz_1 dz_2 = \rho_1 dt, \quad dz_2 dz_3 = \rho_2 dt, \quad dz_1 dz_3 = \rho_3 dt \quad (7)$$

where (4) represents the risk-adjusted process followed by the exchange rate, (5) represents the risk-adjusted process followed by the foreign currency interest rate, (6) represents the risk-adjusted process followed by the domestic currency interest rate, and (7) shows the correlation coefficients between every possible pair of increments to the standard Brownian motions. The process followed by the interest rates show reversion to a long-run mean represented by  $\alpha$  in (5) and by  $\beta$  in (6); the speed of adjustments are represented by  $\kappa$  in (5) and by  $\gamma$  in (6); all the  $\sigma$  parameters represent volatilities and are assumed to be constant.

The implementation of this methodology requires using the following discrete version of the three-factor model described by equations (4) to (7):

$$\frac{S_{t+1} - S_t}{S_t} = (r_L - r_F)\Delta t + \sigma_1 \sqrt{\Delta t} \varepsilon_t \quad (8)$$

$$r_{F_{t+1}} - r_{F_t} = \kappa(\alpha - r_{F_t})\Delta t + \sigma_2 \sqrt{\Delta t} v_t \quad (9)$$

$$r_{L_{t+1}} - r_{L_t} = \gamma(\beta - r_{L_t})\Delta t + \sigma_3 \sqrt{\Delta t} \xi_t \quad (10)$$

$$\text{corr}(\varepsilon_t, v_t) = \rho_1, \quad \text{corr}(v_t, \xi_t) = \rho_2, \quad \text{corr}(\varepsilon_t, \xi_t) = \rho_3 \quad (11)$$

where the disturbances  $\varepsilon_t$ ,  $v_t$ , and  $\xi_t$  present a normal standard distribution and are correlated among them as shown by equation (11). We use the discrete version of the processes followed by the exchange rate and by the interest rates to simulate trayectories of those variables. With those simulations we will be able to compute simulated values for the cash flow the company will have at T. Under stochastic interest rates the cash flow at T for a given simulation j will be computed as:

$$CF_{T,j} = S_{T,j} + \sum_{t=0}^{T-1} {}_t h_{t+1} * \frac{(S_{t+1,j} - {}_t F_{t+1,j})}{B(j,t,T)} \quad (12)$$

where  $B(j,t,T)$  corresponds to the value at t of a zero-coupon bond with maturity at T, in a simulation j. We divide by the value of this zero coupon bond to transform the gains or losses the position will generate in period t to money in period T. Two differences with the non-stochastic scenario already described must be considered here. The first one is the way of we will compute forward prices, and the second one is the procedure we will use to compute the value of the zero-coupon bonds when interest rates are stochastic.

To compute forward prices we can implement the following procedure: First, and following Vasicek (1977) we can use the risk-adjusted-stochastic process presented in equation (6) to obtain the value at t of a one unit of local currency discount bond payable at T,  $B(t,T)$  as:

$$B(j, t, T) = \exp \left[ -r_{L,j} \frac{1 - e^{-\gamma T}}{\gamma} + \frac{\beta \{ (1 - e^{-\gamma T}) - \gamma T \}}{\gamma} - \frac{\sigma_3^2 \{ 4(1 - e^{-\gamma T}) - (1 - e^{-2\gamma T}) - 2\gamma T \}}{4\gamma^3} \right] \quad (13).$$

Second, and following the three factor model proposed by Schwartz (1997), the present value of a forward commitment to deliver one unit of foreign currency at time T,  $P(j, t, T)$ , can be obtained as:

$$P(j, t, T) = S \exp \left[ \frac{-r_f(1 - e^{-\kappa T})}{\kappa} + D(T) \right] \quad (14)$$

where the constant  $D(T)$  is computed as:

$$D(T) = \frac{(\kappa\alpha + \sigma_1\sigma_2\rho_1)\{ (1 - e^{-\kappa T}) - \kappa T \}}{\kappa^2} - \frac{\sigma_2^2 \{ 4(1 - e^{-\kappa T}) - (1 - e^{-2\kappa T}) - 2\kappa T \}}{4\kappa^3} \quad (15).$$

Finally, we can generate the forward price we are referring to in equation (12) by dividing the present value of a forward commitment to deliver one unit of the foreign currency at time T computed through equation (14), by the present value of a unit discount bond payable at T computed through equation (13), as it is shown next:

$$F(j, t, T) = \frac{P(j, t, T)}{B(j, t, T)} \quad (16).$$

From here we can obtain an optimal long-term hedging strategy using short-term forward contracts. The optimal hedging strategy H will result from finding those  $h_{t+1}$  for  $t=0$  to  $t=T-1$  who minimizes the variance of the cash flows the firm will receive at T. The procedure requires to generate those cash flows described in (12) through simulation, to compute their variance, and finally to find the strategy H that minimizes the variance.



#### 4. Implementing the Simulation-Based Optimal Hedging Strategy

##### A. The Base Case

In this section I implement the described methodology. The values for the parameters required by the model used in the base case to be considered are summarized in Table 1, and correspond to parameters computed using data on the Chilean peso to American dollar exchange rate and on the corresponding interest rates in each currency<sup>3</sup>. The data used to generate these parameters were obtained from the Chilean Central Bank and correspond to the 1998 to 2001 period. With the parameters described here the methodology is implemented to find the optimal hedging strategy and to measure the quality of the strategy as described in section 3.

**Table 1**  
*Parameters for the Chilean Peso/American Dollar Case*

	Exchange Rate	Local Interest Rate	External Interest Rate
Initial Values	700,0	0,02	0,015
Standard Deviation	0,035	0,050	0,010
Kappa			0,5312
Alpha			0,0160
Gamma		0,6500	
Beta		0,0218	
Corr(TC,i)	1,00	0,15	0,10
Corr(LIR,i)	0,15	1,00	0,14
Corr(EIR,i)	0,10	0,14	1,00

The exchange rate is in Chilean pesos per American dollars.

<sup>3</sup>An interesting point to discuss is whether the firm should try to hedge the nominal exchange rate or the real exchange rate. While the nominal exchange rate would be the one we want to hedge in those cases when the investment horizon is short or medium term, an argument can be given to say that the exchange rate we want to hedge in the very long term would be the real one, given the empirical evidence that relative purchasing power parity would tend to hold in the long run. The methodology proposed here would handle both problems with the same efficiency. In the particular example developed here it is assumed the investor is not looking at the very long run and that is the reason to hedge the nominal exchange rate.

Table 2 shows the results of hedging a long run cash flow in American dollars to be received five years from now using a combination of positions in short run forward contracts. Here it is assumed that those contracts have a maturity of six months. We also assume that the positions are reviewed every six months, until the five years investment horizon is reached. Table 2 outlines the optimal hedging positions to be taken every six months. It shows that for the first six months we should short 0.7984 US\$ in forward contracts, and that six months later we should short 0.8536 US\$ in forward contracts once the original position has expired. Table 2 also shows that six months before the expiration of the five years time horizon we should short 1.0266 US\$ in forward contracts. This optimal hedging strategy would allow the investor to eliminate 91.86% of the five years cash flow volatility<sup>4</sup>.

**Table 2**  
*Optimal Hedging: The Basic Case*

0h <sub>1</sub>	0,7984
1h <sub>2</sub>	0,8536
2h <sub>3</sub>	0,8815
3h <sub>4</sub>	0,9230
4h <sub>5</sub>	0,9580
5h <sub>6</sub>	1,0205
6h <sub>7</sub>	1,0162
7h <sub>8</sub>	1,0083
8h <sub>9</sub>	1,0501
9h <sub>10</sub>	1,0266

Variance of Cash Flow without hedging:	26,42
Variance of Cash Flow with hedging:	2,15
% of Hedging Reached:	91,86%

This table shows the optimal hedging positions to be taken in six-month forward contracts.

In the next subsection I explore how sensitive is the solution found here to

<sup>4</sup>As explained before, the volatility of the cash flow without hedging corresponds to the variance of the cash flow 5 years from now if we have one long position in the foreign currency, and the variance of the cash flow with optimal hedging results from finding the combination of position in forward contracts that would minimize the variance of the cash flow resulting from investing in the foreign currency and taking positions in six-month forward contracts.

In the next subsection I explore how sensitive is the solution found here to changes in some of the key parameters used to describe the initial scenario. In particular we will see how the proportion of total volatility hedged depends on the volatility of the exchange rate, on the volatility of the interest rates, and on the degree of correlation among some of these variables. We will also look at how sensitive to these parameters are the hedging strategies identified previously.

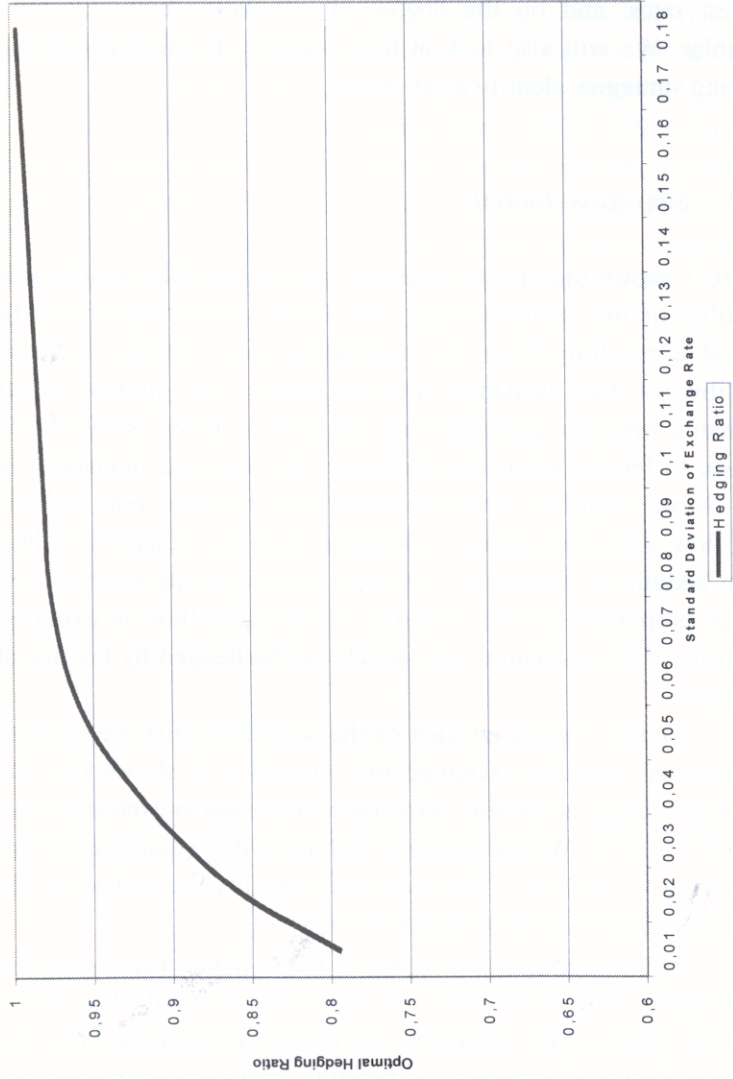
### *B. Sensitivity Analysis*

Figure 1 shows the optimal hedging ratio achievable, as a function of the volatility of the exchange rate. The figure shows that the hedging ratio reachable resulting from the optimal hedging strategy increases as the volatility of the exchange rate increases. The optimal hedging ratio reachable when the exchange rate standard deviation is 0.01 is 79.0%, and it goes to 99.0% when the exchange rate standard deviation increase to 0.18. More volatility in the exchange rate increases both volatility of the cash flow when no hedging is attempted, and also volatility of the cash flow when hedging is attempted, but when exchange rate volatility increases, a bigger proportion of the volatility of the cash flow is explained by the volatility of the exchange rate, which can be hedged by the use of forward contracts.

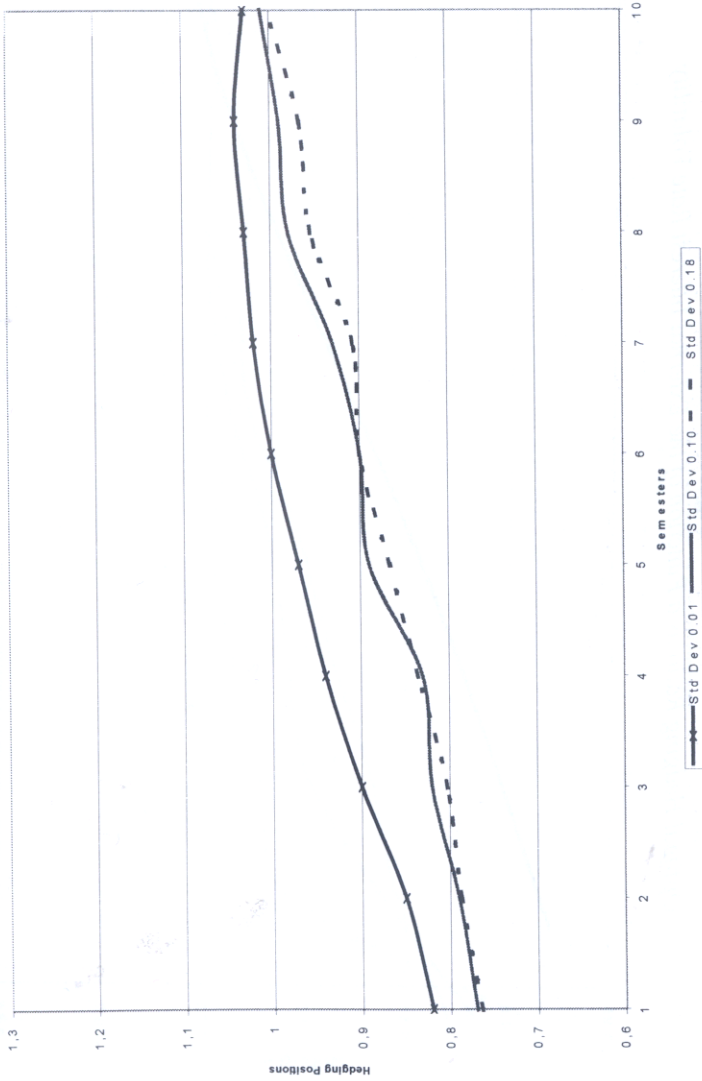
Figure 2 gives an idea of the sensibility that the optimal hedging strategy has towards exchange rate volatility. It shows that the optimal hedging strategy is not very sensitive to a change in exchange rate volatility from 0.1 to 0.18. We see a greater reaction when volatility of exchange rate goes from 0.1 to 0.01. The lower the volatility the higher are the hedging positions composing the hedging strategy

Figure 3 shows the impact of the external interest rate volatility in the hedging ratio achievable. As expected, we verify that the higher the external interest rate volatility, the lower the hedging ratio that we can achieve. Figure 4 presents the hedging positions composing the optimal hedging strategy over time, for different degrees of volatility in the external interest rate. The figure suggests that the hedging positions increase as the volatility of the external interest rate increases.

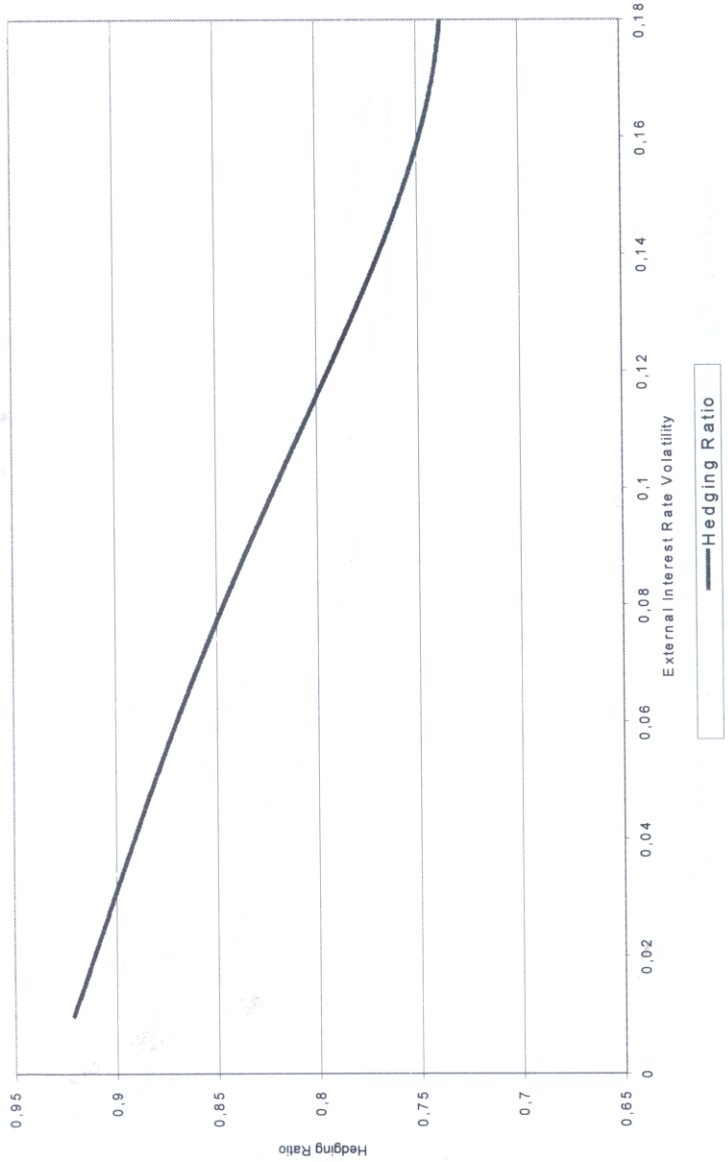
**Figure 1**  
*Optimal Hedging Ratio as a Function of the Exchange Rate Volatility*



**Figure 2**  
*Hedging Positions Over Time as a Function of Exchange Rate Volatility*



**Figure 3**  
*Optimal Hedging Ratio as a Function of the External Interest Rate Volatility*



**Figure 4**  
*Hedging Positions Over Time as a Function of Local Interest Rate Volatility*

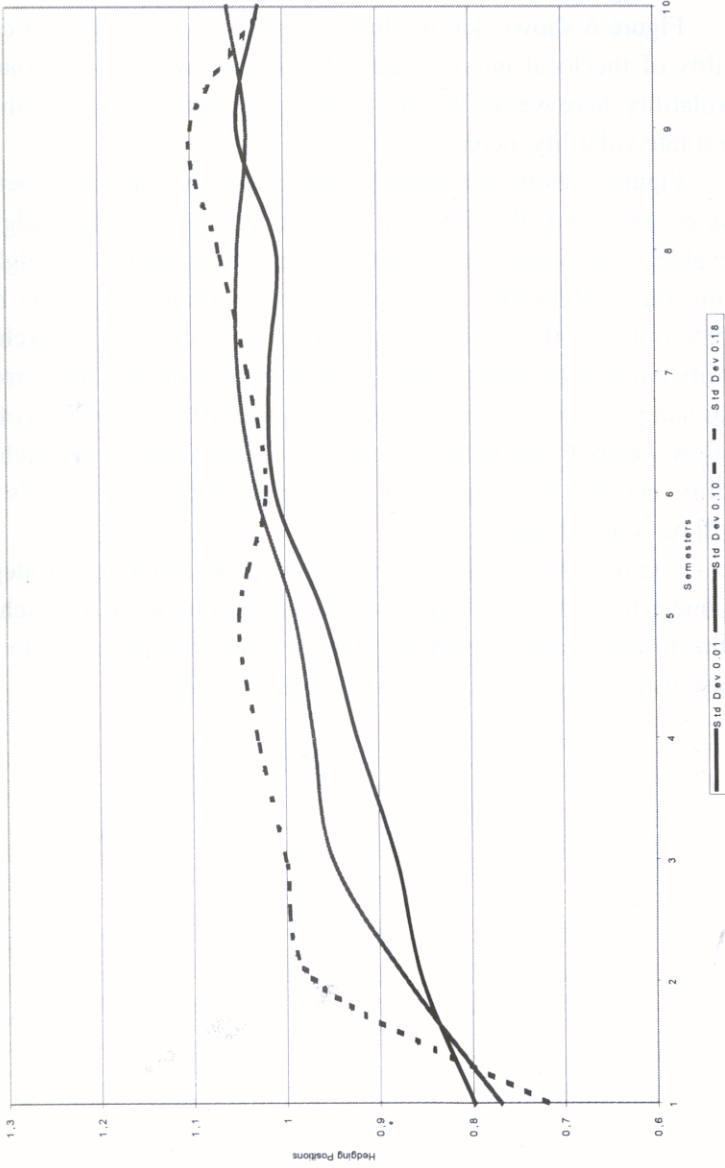


Figure 5 shows the impact of the internal interest rate volatility in the hedging ratio achievable. The higher the internal interest rate volatility, the lower the hedging ratio achievable.

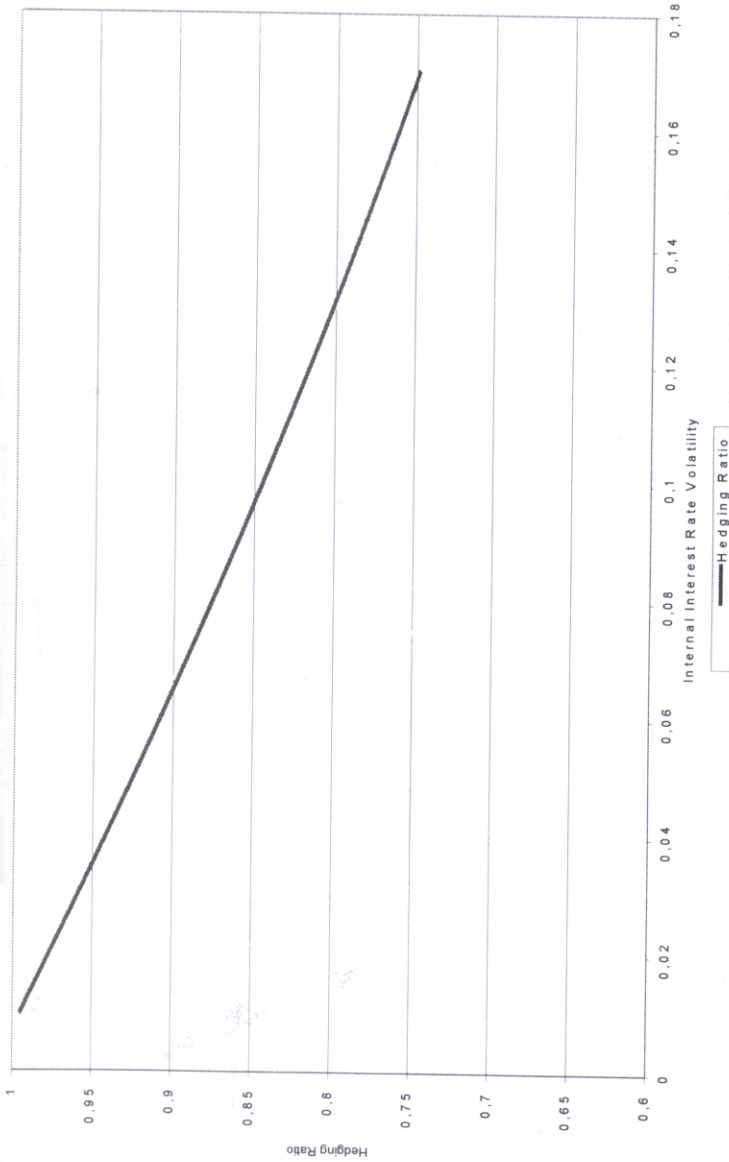
Figure 6 shows optimal hedging strategies for different degrees of volatility of the local interest rate volatility. As with the external interest rate volatility, here we see that the hedging positions increase as the internal interest rate volatility increases.

Figure 7 shows the impact of the degree of correlation between the exchange rate and the internal rate of interest in the hedging ratio achievable. The higher the degree of positive correlation the higher is the hedging ratio achievable. This makes sense because as it has been said before with the methodology proposed here we can hedge the exchange rate volatility but not the interest rates volatilities. The higher the correlation of the exchange rate to the interest rates the higher the proportion of the total cash flow we are trying to hedge that will be explained by the exchange rate volatility, and the higher then the proportion of that cash flow's volatility we will be able to hedge.

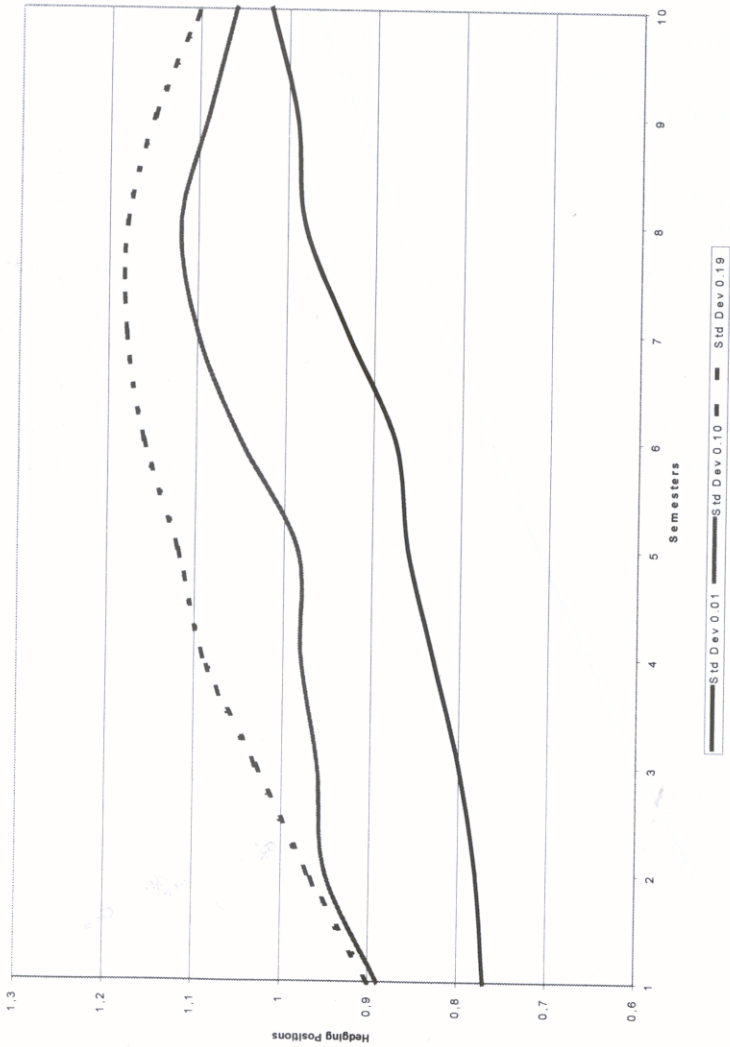
Figure 8 shows the evolution of optimal hedging strategies over time, under three different degrees of correlation between the exchange rate and the internal rate of interest. Here we see that positions to be taken increase the closer to 1 the correlation coefficient is.



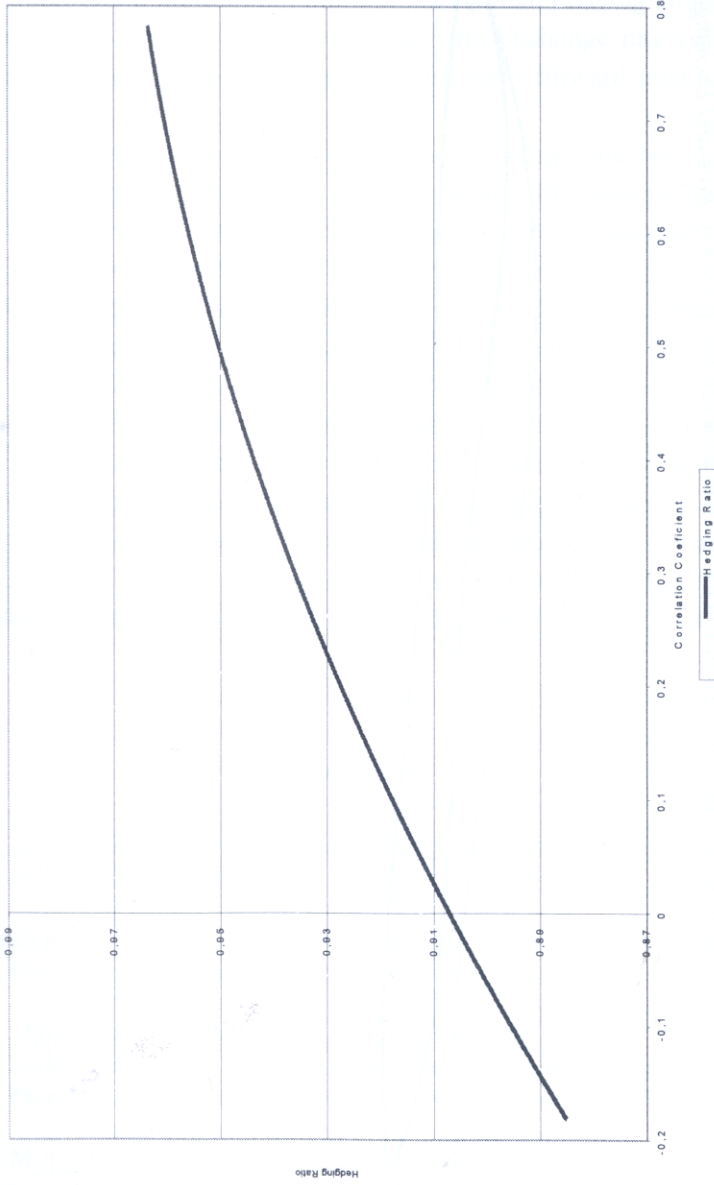
**Figure 5**  
*Hedging Ratio as a Function of Internal Interest Rate Volatility*



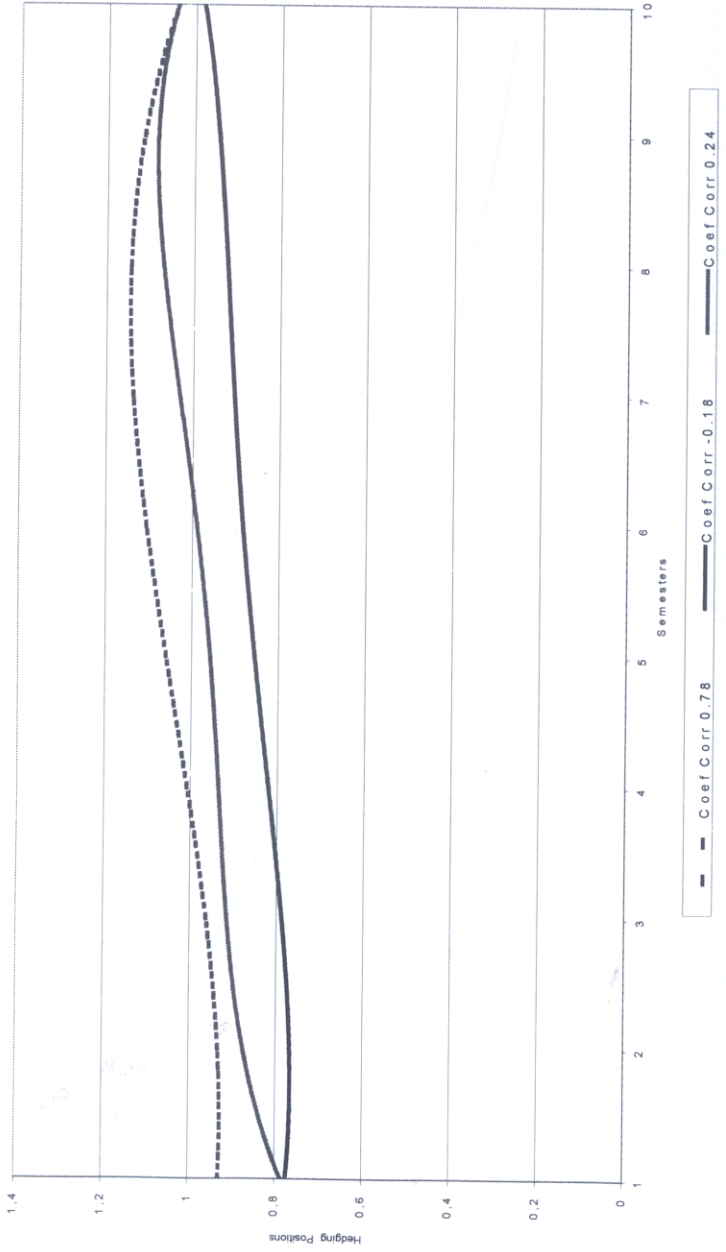
**Figure 6**  
*Hedging Positions Over Time as a Function of Local Interest Rate Volatility*



**Figure 7**  
*Hedging Ratio as a Function of the Correlation Coefficient Between Internal Interest Rate and Exchange Rate*



**Figure 8**  
*Hedging Positions Over Time as a Function of Correlation Coefficient Between Local Interest Rate and Exchange Rate*



## Summary and Conclusions

This paper presents a methodology to find the optimal hedging strategy to be followed by an investor exposed to long term exchange rate volatility under a situation where he can only use short term forward contracts to hedge.

Given an scenario where not only the exchange rate but also the internal and external interest rates are stochastic, the optimal hedging strategy is defined as the one that allows to minimize the variance of the long term cash flow the investor will receive after T years.

The methodology is described and a few results are presented based on the Chilean peso- American dollar exchange rate case. It is shown that more than 90% of the variance of a five years cash flow can be eliminated using the appropriate hedging strategy. The impact that the values of some key parameters has in the quality of the hedging strategy is also reviewed. We find that the quality of the hedging strategy would increase the higher are the volatility of the exchange rate and the correlation between exchange rate and interest rates. We also find that the quality of the hedging strategy decreases as the volatility of the interest rates increases.

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