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Risk-Based Premiums for Pension Insurance

Salvador Zurita L.
Universidad de Chile

Extracto

La Corporación de Garantía de Beneficios de Pensiones garantiza las pensiones de beneficio definido en Estados Unidos, cobrando una prima a cambio (fija después de un desfinanciamiento dado). En este artículo se desarrolla un modelo para estimar las primas que debiera cobrar dicha corporación para reflejar el valor económico del seguro que provee. El modelo utiliza la analogía entre seguro de pensiones y opciones de venta financieras para derivar ecuaciones de valoración y sus soluciones analíticas. También incluye auditorías periódicas aleatorias que siguen un proceso Poison, cuya frecuencia promedio puede ser elegida por la autoridad económica con el objetivo de alcanzar optimalidad paretiana. El modelo se estima para una muestra de firmas norteamericanas en el período 1982-1986. La principal implicancia de política es que las primas de seguro basadas en riesgo varían a tasa creciente con el nivel de desfinanciamiento del fondo de pensiones, en contraste con la legislación entonces vigente.

Abstract

The present paper develops a model to compute risk-based premiums for the USA pension insurance administered by the public Pension

This paper draws on my Dissertation at UCLA. *I am indebted to my Ph.D. Committee for guidance: Nathaniel Grossman, Walter N. Torous, J. Fred Weston, N. Donald Ylvisaker, and especially Eduardo S. Schwartz, the Chairman. Of course the usual disclaimer applies.*

Benefit Guaranty Corporation (PBGC). Pension insurance is shown to be analogous to a financial put option, and pricing equations and their analytical solutions are obtained. The model includes costly audits that follow a Poisson process, whose average frequency is determined by the policymaker in order to attain Pareto-optimality. The model is estimated for a sample of US firms for the period 1982-1986. The main policy implication is that risk-based premium rates increase at an increasing rate with the level of underfunding, in contrast with the current law of flat premium rates after certain level of underfunding.

Introduction[1]

Under the Employee Retirement Income Security Act (ERISA) of 1974 all qualified defined benefit plans are required to participate in the insurance program administered by the public sector Pension Benefit Guaranty Corporation (PBGC), in order to protect participants and beneficiaries against loss of benefits in case of termination of the pension plan.

The PBGC guarantees the participants' basic pension benefits up to the maximum permitted by law for the year in which termination occurs.[2] In order to meet this obligation, the PBGC is authorized to charge a premium per participant in a plan, which is determined by Congress. Firms are constrained in the funding levels of their pension funds by ERISA, which requires a minimum funding level, and by the Internal Revenue Service (IRS), which sets a maximum funding level. Also, if a Defined Benefit pension plan is terminated, firms are liable up to 30% of their net worth to the PBGC.

Today the PBGC covers some 40 million American workers and retirees by insuring about 112,000 private sector pension plans (see PBGC 1991a). It administers two pension insurance programs: the single-employer program and the multi-employer program. The former protects approximately 31 million participants in some 93,000 single-employer pension plans; the latter protects about 8.4 million participants in about 2,300 plans.[3] Multi-employer pension

[1]In the empirical part of this paper we consider the legislation relevant for the sample period.

[2]In 1991 the maximum was $2,250.00 per month for a participant in a single-employer plan who retires at age 65 with no survivor benefits (see PBGC 1991a).

[3]The PBGC protects the retirement incomes of nearly 40 million participants in more than 95,000 private-sector defined benefits pension plans.

plans are maintained under collectively-bargained agreements between employee representatives and two or more unrelated employers.

Table 1 shows the PBGC's claims experience with single-employer plans, since its first year of operations until 1990. Despite the decline in the number of plans terminated over the last few years (1985-1990), the net losses from plan terminations have increased dramatically (67% in the 1985-1990 period with respect to 1980-1984). Moreover, according to PBGC estimations the agency faces expected losses of $1.1 billion for 34 plans that are expected to terminate in 1991. The guaranteed liabilities in those 34 plans are more important than the liabilities of about 1,600 plans terminated in the 16 years of PBGC's existence. This shows the tremendous sensitivity of the PBGC's financial situation to terminations of very large underfunded plans.[4]

Table 1

Claims Experience from Single-Employer Plans
(Dollars in Millions)

YEAR OF TERMINATION	NUMBER OF PLANS	BENEFIT LIABILITY	TRUST PLAN ASSETS	RECOVERIES FROM EMPLOYERS	NET LOSSES
1975-1979	585	402	151	59	192
1980-1984	599	1,270	512	136	622
1985-1990	374	2,063	469	385	1,209
TOTAL TERMINATED	1,558	3,736	1,132	580	2,024
PROBABLE (*)	34	4,345	2,284	950	1,111
TOTAL	1,592	8,081	3,416	1,530	3,135

*Plans whose termination is being negotiated and that the PBGC estimates will be terminated during 1991.

Source: PBGC (1991b).

[4]According to figures by the PBGC (1991b), there is between $20 and $30 billion of underfunding in pension plans concentrated in the steel, airline, and automobile industries. This amounts to about 10 times the PBGC's annual premium income.

Table 2 shows the trends of terminated plans, including information about the funding level, the net losses as a percentage of guaranteed benefit liabilities, and the average net loss per terminated plan. The funding level during the last five years has decreased, resulting in higher net losses as a percentage of guaranteed benefits. Since the plans terminated over the last five years have been considerably larger, the average net loss ($ 3.2 million) is almost three times as big as the average for the 80-84 period and ten times the average loss for the period 75-79.

Table 2

Trends of Terminated Plans

YEAR OF TERMINATION	FUNDING LEVEL	NET LOSSES AS A PERCENT OF GUARANTEED BENEFIT LIABILITY	AVERAGE NET LOSS PER TERMINATED PLAN (DOLLARS IN MILLIONS)
1975-1979	37%	48%	$0.3
1980-1984	40%	49%	$1.0
1985-1990	23%	59%	$3.2

Source: PBGC (1991b).

When the PBGC was first instituted in 1974, firms were charged a fixed premium of $1.00 per employee per year for pension insurance. In 1978, the PBGC raised the premium to $2.60 per employee per year. In 1986 the PBGC further increased the premium rate to $8.50. Although the Multi-Employer Pension Plan Amendment Act of 1980 directed the PBGC to consider the possibility of a graduated premium rate based on risk, the premium rates charged continued to be fixed until 1988. Effective January 1, 1988, the premium rates for the single employer program were increased to $16 per participant per year, plus a variable rate of up to $34 per participant based on $6 per $1,000 (or fraction thereof) of unfunded vested benefits.

In 1990 Congress enacted an increase in the premium for single-employer plans as part of the budget agreement for fiscal year 1991. As a result, in 1991 the basic premium increased to \$19 per participant. To increase funding incentives, the additional variable rate for underfunded plans was also increased to \$9 per \$1,000 of unfunded vested benefits, capped at \$53 per participant for a total maximum premium of \$72 per participant (see PBGC 1991b).

The purpose of the present paper is to obtain a premium rate schedule for the PBGC insurance based on the economic risks of the different pension plans. The option pricing framework provides a useful approach to this problem. The insurance provided by the PBGC is analogous to an exchange option: an insured firm has the option to transfer its pension liabilities to the PBGC in return for only the pension assets and 30% of the value of the firm. Notice that this option involves changing one risky asset (the pension liabilities) for another (the pension assets plus 30% of the firm's equity), a problem that was originally studied in Margrabe 1978.

This correspondence between put options and term insurance policies has long been recognized in the literature. For example Merton 1977, 1978 used the option pricing framework to value deposit insurance, while Cummins 1988 used it to value insurance guarantee funds in general.

Several authors have used the option pricing methodology to analyze the insurance provided by the PBGC (see for example Bulow 1982; Marcus 1985, 1987; Sharpe 1976; Treynor 1977; da Motta 1979, and Langetieg et al 1982). However, most of these papers are concerned with the economic effects of ERISA, and not with deriving a valuation formula for the PBGC insurance. An exception is Marcus 1985, who obtains estimates of the present value of the PBGC insurance liabilities (but not premium rates). The contribution of the present paper is to provide risk-based *premium rates* for PBGC insurance, in a setting where the PBGC audits every so often a firm's pension plans.[5]

The present paper extends Merton's 1978 model for bank insurance in three respects: 1) Our model deals with the computation of insurance premium rates rather than with present value liabilities; 2) the stochastic processes for assets and liabilities and the boundary conditions are re-specified in order to apply them to pension plans rather than to banks, which results in a stochastic rather than a deterministic exercise price for the put, and 3) the auditing process

[5]Marcus 1985 does not include audits. But it is apparent that periodical audits will be required in any sensible insurance program, since otherwise the potential losses to the PBGC would be enormous.

is endogenously designed, in order to obtain Pareto-optimal properties in the insurance provided by the government.

Section I presents two models of pension insurance; section II contains some empirical estimates of insurance premiums for a sample of US firms; section III discusses optimal auditing frequency, and section IV concludes the paper.

I. Models of Pension Insurance

We model the problem of computing risk-based premium rates for a pension fund in a world where trading takes place continuously. It is assumed that there is a unique instantaneous interest rate at which borrowing and lending take place, and that the intertemporal capital asset pricing model (ICAPM) holds.[6]

The PBGC enters into a contractual agreement with a firm in order to insure its defined benefit pensions for a premium. The premium rate is defined as the rate of payments per unit time that the firm has to pay to the PBGC in exchange for pension insurance. Since the PBGC is expected to finance itself, we are interested in determining actuarially fair premium rates, i.e. the premium rates that give zero expected profits to the PBGC, accounting for the risks involved and the auditing costs (which are specified later on).

The PBGC conducts random audits to verify the solvency of the different pension funds that it insures. The audits are costly. A feature of the model is that even though these surveillance costs are ex post paid by the PBGC, they are paid ex ante by the firms, since the expected costs of the insurance system are included in the insurance premium rate.

We assume that the assets backing the pension fund follow a diffusion process:

$$d\,A = [\alpha_A\, A + \eta_A\, L\, \delta\, L - g\, L]\, dt + A\, \sigma_A\, d\, z_A, \qquad (1)$$

[6]This last assumption is not crucial. As it will become clear later, we only need that the jump part corresponding to the random audits be not priced in equilibrium.

where

A	=	The assets backing the pension fund (pension assets plus 30% of the value of the firm, under the current legislation)
L	=	The pension liabilities (accrued benefits)
α_A	=	The instantaneous expected rate of return on the existing assets per unit time
η_A	=	The instantaneous rate of contributions by the firm to the pension fund as a fraction of the pension liabilities per unit time
δ	=	The instantaneous rate of payments to retirees as a fraction of pension liabilities per unit time
g	=	The premium rate per unit of pension liabilities
σ_A	=	The instantaneous standard deviation of the return on the assets
$d\, z_A$	=	Increment to a Gauss-Wiener process

Equation (1) says that the value of the assets backing the pension fund increases due to a normal rate of return and as the firm contributes new funds. It decreases as the pension funds are used to pay retired workers and the insurance premium is paid to the PBGC. Since there is uncertainty concerning the return on the assets, there is a stochastic part in (1). With this specification we abstract from problems of moral hazard by assuming that the rate of contributions by the firm to the pension plan is held constant and we focus on the risks that come from market fluctuations in pension assets and liabilities.[7] Including the possibility of strategic behavior on the part of the firm in its contributions to the pension plan would undoubtedly increase the actuarially fair pension premiums.

The pension liabilities consist of the accrued benefits (the present value of pension payments) to insured workers. We represent them by a portfolio of bonds that have coupon payments and maturities that match the payments of the pension fund. The value of the bond portfolio is assumed to follow the diffusion

$$d\,L = [\alpha_L\, L + \eta_L\, L\ \delta\ L]\ dt + L\ \sigma_L\ d\,z_L, \qquad (2)$$

[7]We relax this assumption in Section IV, where we allow the audit frequency to affect the firm's rate of contributions to its pension plan.

where

α_L The instantaneous growth rate of existing liabilities per unit time
η_L The instantaneous increase in liabilities from new workers coming into the firm and from a greater length of employment of old workers
σ_L The instantaneous standard deviation of the pension liabilities
dz_L An increment to a Gauss-Wiener process

Equation (2) shows that pension liabilities increase with their normal growth rate, length of employment and the number of insured workers. The accrued benefits decrease as pensions are paid to retirees. There is a stochastic component since interest rates are stochastic.

The assumption that the parameters of the model are time independent restricts our attention to the steady state of a well established pension plan. For this reason, the present value of pension assets and liabilities has to be bounded as time increases and the pension plan surplus has to converge to zero as time goes to infinity. These equilibrium conditions impose restrictions on the parameters: $\delta - \eta_L \geq 0$ and $\delta + g - \eta_A \geq 0$.[8]

The two stochastic processes are related by the correlation coefficient

$$d\, z_A\, d\, z_L = \rho\, dt. \tag{3}$$

For simplicity we assume that the auditing costs are a constant proportion of the liabilities in the pension fund:

$$C\,(L) = c\, L. \tag{4}$$

We consider two models of pension insurance, depending on whether the premium rates are revised after each audit or not. In the first model it is assumed that after each audit the PBGC will decide on a new insurance premium; in the second one the premium rate is set at the beginning of the pension plan insurance contract, and is held constant thereafter for the entire life time of the pension plan. In the latter model the objective of an audit is to determine

[8]The proof is available from the author.

whether the pension plan should be allowed to continue in existence (if financially sound) or should be terminated (otherwise). Both types of insurance seem to be offered in different industries; in the automobile industry insurance contracts seem closer to the first model, while in the medical industry they seem closer to the second one. The pension insurance currently in operation is of the first type, with premium rates set once a year.

A. PREMIUM RATES REVISED AFTER EACH AUDIT

In the first model the PBGC evaluates the financial situation of a pension plan after each audit, and determines the premium rate that the company must pay until the next audit. The coverage period is random. More specifically, the audits are assumed to follow a Poisson process with characteristic parameter λ.

Since after this period's insurance an entirely new premium will be optimally determined, this case is conceptually equivalent to a one shot situation where the insurance contract terminates after one (random) period.

We are interested in obtaining a fair premium rate for the insurance provided by the PBGC (rate g in our model). Let $P(A,L)$ be the present value of pension insurance (i.e., the value of the pension put created by the insurance offered by the PBGC). As usual, we assume that $P(.)$ is a twice differentiable function of pension assets and liabilities.[9]

Note that $P(.)$ is not dependent on the time to maturity. In our model the time remaining to audit is stochastic, and therefore unknown. Moreover, since the interval of time between the initiation of the insurance contract and the audit is exponentially distributed, the time elapsed since the start of the insurance contract does not give any information as to when the next audit will be conducted.[10] Hence, the present value of pension insurance cannot depend on time.

[9]Of course this assumption will be verified in the solution later on.

[10]An important property of exponentially distributed random variables is that if X is such a variable, then $P(X > a+b \,/\, X > a) = P(X > b)$. In plain words, if X represents the time to the audit by the PBGC, then conditioned on there having been no audit by time a, the probability of having an audit in the next b units of time is equal to the unconditional probability of no audit during the first b units of time. This implies that the time elapsed since the pension plan was started will neither increase nor decrease the probability of a new audit in a given length of time.

From an analytical perspective, our pension put can be decomposed in two components: 1) the value of the put option to exchange one risky asset for another, and 2) the surveillance costs, which will be incurred at the end of the coverage period. Let dP represent the change in value of the pension put. With probability $(1 - \lambda\, dt)$ there will not be any audit in the next instant of time and, since the PBGC liability is a twice differentiable function of pension assets and liabilities, we can use Ito's Lemma (see, for example, Ingersoll 1987, pp. 347-48) to obtain

$$dP - gL\,dt = \phi\,[\,P(A,L)\,]\,dt + A\,\sigma_A \frac{\partial P}{\partial A}\,dz_A + L\,\sigma_L \frac{\partial P}{\partial L}\,d\,z_L, \tag{5}$$

where the operator ϕ is defined by

$$\phi = \frac{1}{2}\,[A^2\,\sigma_A^2 \frac{\partial^2}{\partial A^2} + L^2\,\sigma_L^2 \frac{\partial^2}{\partial L^2} + 2\,A\,L\,\sigma_A\,\sigma_L\,\rho \frac{\partial^2}{\partial A\,\partial L}] \tag{6}$$

$$+ [\alpha_A\,A + \eta_A\,L - \delta\,L - g\,L] \frac{\partial}{\partial A} + [\alpha_L + \eta_L - \delta]\,L \frac{\partial}{\partial L} - g\,L.$$

This operator captures the deterministic component of the total change in the value of the pension put. A peculiar feature of the model is that the premium rate payments gL are included with a negative sign. These payments play the role of coupon payments in the case of bondholders, with the difference that they are paid -not received- by the firm that holds the pension put (in order to keep it).

Since the time to the audit and the end of the insurance contract is exponentially distributed, with probability $\lambda\, dt$ there will be an audit at the next instant of time and there will be a jump in the return to the pension insurance. There are two cases to consider:

Case 1. Audit and pension plan solvent ($A \geq L$).

If an audit is conducted and the pension plan is found solvent, then the value of the pension insurance increases in the amount of the audit cost, but it also decreases in an amount P since the insurance contract terminates:

$$\Delta P = C(L) - P. \tag{7}$$

Case 2. Audit and pension plan insolvent ($A < L$).

If an audit is conducted and the pension plan is found insolvent, then the value of the pension insurance increases by the audit costs and by the difference between pension liabilities and pension assets (L-A). Also, the pension insurance contract is fully terminated, which makes the old put worthless (-P). The total change in the value of the pension put is

$$\Delta P = C(L) + L - A - P. \tag{8}$$

Let α_P represent the gross return on the pension put. Since the probability of an audit over the next instant of time is $\lambda\, dt$, the expected total change on the value of the pension put per unit time, $P\, \alpha_P$, including premium payments, can be computed from (6)-(8), as follows:

Case 1. Pension plan solvent ($A \geq L$):

$$P\ \alpha_P = \phi\ [P(A,L)] + \lambda\ [C(L) - P\ ]. \tag{9}$$

Case 2. Pension plan insolvent ($A < L$):

$$P\ \alpha_P = \phi\ [P(A,L)] + \lambda\ [C(L) + L - A - P\ ]. \tag{10}$$

Assuming that the ICAPM holds, the pension put, the pension assets and the pension liabilities should lie on the security market line:

$$\alpha_i - r = \beta_i\ (\alpha_M - r), \tag{11}$$

where r represents the instantaneous risk-free interest rate, α_M stands for the expected return on the market portfolio, β_i represents the systematic risk of security i, and the subindex i can take the values P, A and L, representing the equilibrium condition for the pension put, the pension assets, and the pension liabilities, respectively.

From (5), if the probability of an audit is uncorrelated with the return on the market, the systematic risk of the pension put $P(.)$ is

$$\beta_P = [\frac{A}{P}]\frac{\partial P}{\partial A}\ \beta_A + [\frac{L}{P}]\frac{\partial P}{\partial L}\beta_L. \tag{12}$$

Combining (11) and (12) it follows that in equilibrium

$$P\ [\alpha_P - r] = A\ \frac{\partial P}{\partial A}\ [\alpha_A - r] + L\ \frac{\partial P}{\partial L}\ [\alpha_L - r]. \tag{13}$$

The final step to obtain an equilibrium pricing equation is to substitute the expected returns on the pension put (9)-(10) in the required expected returns (13). As before, there are two cases to consider:

Case 1. Pension plan solvent ($A \geq L$):

$$\frac{1}{2}\ [A^2\ \sigma_A^2\ \frac{\partial^2 P}{\partial A^2} + L^2\ \sigma_L^2\ \frac{\partial^2 P}{\partial L^2} + 2\ A\ L\ \sigma_A\ \sigma_L\ \rho\ \frac{\partial^2 P}{\partial A\ \partial L}] \tag{14}$$
$$+ [\eta_A\ L - \delta\ L - g\ L + A\ r]\ \frac{\partial P}{\partial A} + [\eta_L - \delta + r]\ L\ \frac{\partial P}{\partial L}$$
$$- (\lambda + r)\ P + \lambda\ c\ L - g\ L \quad = \quad 0.$$

Case 2. Pension plan insolvent ($A < L$):

$$\frac{1}{2}\ [A^2\ \sigma_A^2\ \frac{\partial^2 P}{\partial A^2} + L^2\ \sigma_L^2\ \frac{\partial^2 P}{\partial L^2} + 2\ A\ L\ \sigma_A\ \sigma_L\ \rho\ \frac{\partial^2 P}{\partial A\ \partial L}] \tag{15}$$
$$+ [\eta_A\ L - \delta\ L - g\ L + A\ r]\ \frac{\partial P}{\partial A} + [\eta_L - \delta + r]\ L\ \frac{\partial P}{\partial L}$$
$$- (\lambda + r)\ P + \lambda\ (c\ L + L - A) - g\ L \quad = \quad 0.$$

Equations (14-15) are partial differential equations. It is possible to transform them to ordinary differential equations by letting $x = A/L$ represent the ratio of pension assets to pension liabilities and postulating that the pension put is linearly homogeneous,[11] i.e., that the value of the pension put per unit liability $p = P/L$ depends on the pension assets and liabilities only through the ratio x (i.e., $p = p(x)$).

Substituting $p(x)$ in (14-15) and defining the volatility of the ratio x by

$$\sigma^2 \quad = \quad \sigma_A^2 + \sigma_L^2 - 2\ \sigma_A\ \sigma_L\ \rho, \tag{16}$$

[11] Of course, we will check that this assumption is valid.

we obtain the following equations:

Case 1. Pension plan solvent ($x \geq 1$):

$$\frac{\sigma^2}{2} x^2 \frac{d^2 p}{dx^2} + [(\delta - \eta_L) x - (\delta + g - \eta_A)] \frac{dp}{dx} - (\lambda + \delta - \eta_L) p - g + \lambda c = 0. \tag{17}$$

Case 2. Pension plan insolvent ($0 \leq x < 1$):

$$\frac{\sigma^2}{2} x^2 \frac{d^2 p}{dx^2} + [(\delta - \eta_L) x - (\delta + g - \eta_A)] \frac{dp}{dx} - (\lambda + \delta - \eta_L) p - g + \lambda (c + 1 - x) = 0. \tag{18}$$

Since neither A nor L appear in (17-18) except in the form of the ratio $x = A/L$, our previous conjecture of homogeneity is confirmed as promised in footnote 8. Note also that the interest rate does not appear in the pricing equations, which is consistent with Margrave 1978.[12] Let us denote by $p_+(x)$ the solution to (17) and by $p_-(x)$ the solution to (18).

Equations (17-18) are subject to the following boundary conditions:[13]

1. Continuity of the pension put function,

$$p_-(1) = p_+(1). \tag{19}$$

2. Continuity of the first derivative,

$$\frac{dp_-}{dx}(1) = \frac{dp_+}{dx}(1). \tag{20}$$

[12]The intuition for this result is that the exercise price of the option need not be discounted at the rate r, being a stochastic value itself. It is due to this nice feature of the pricing equation that we do not need to assume constant short term interest rates.

[13]Note that we do not require that the put be non-negative, since for g large enough it will be negative. In fact, we will impose that the put be zero in expected value, which implies that with nonzero probability ex post it will be negative. A proof of boundary conditions 3 and 4 is available from the author.

3. Value of p at zero.

$$p_{-}(0) = 1 + c. \tag{21}$$

4. Value of p at infinity,

$$p_{+}(x) \text{ is bounded as } x \to \infty. \tag{22}$$

The first two conditions say that both the value of the put and the hedging ratio (which is its first derivative) are continuous functions of the pension asset to pension liability ratio. The intuition for the third condition is that when the pension asset to pension liability ratio becomes zero, the pension plan will have to stop paying the insurance premium to the PBGC and the pensions to the retired workers. In this situation it is going to be evident that it is having financial difficulties, so that an audit will be conducted with probability one, and as a result the pension plan will be terminated. Since there are no assets, the PBGC will have to cover the full value of the pension liabilities (L) and the audit cost (cL). The intuition or condition 4 is that as the pension asset to pension liability ratio becomes very large, the pension plan becomes very safe, so that the pension put will consist basically of audit costs. These audit costs are a constant proportion of the pension liabilities, whose present value is bounded, so that the pension put per unit of liability has to be bounded as the asset to liability ratio increases.

Fortunately, these differential equations have an analytical solution. Let $p_{+}(x)$ represent the value of the put per unit of liability for a solvent pension plan (i.e., the solution to (17)) and $p_{-}(x)$ the value for an insolvent one (i.e., the solution to (18)). Then,

$$p_{-}(x) = -x + \frac{\delta - \eta_A + \lambda(c+1)}{\lambda + \delta - \eta_L} + \tag{23}$$

$$a_1 e^{-\frac{m}{x}} x^{-\frac{\gamma - q}{2}} M\left(\alpha, \gamma, \frac{m}{x}\right) + a_2 m^{1-\gamma} e^{-\frac{m}{x}} X^{\frac{\gamma - 1}{2}} M\left(\alpha - \gamma + 1, 2 - \gamma, \frac{m}{x}\right),$$

and

$$p_{\cdot}(x) = \frac{\lambda c - g}{\delta + \lambda - \eta_L} + e^{-\frac{m}{x}} x^{-\frac{(\gamma - q)}{2}} b_1 M(\alpha, \gamma, \frac{m}{x}), \tag{24}$$

where $M(.)$ represents the Kummer confluent hypergeometric function. The new parameters are defined by

$$l = \frac{2(\delta - \eta_L)}{\sigma^2}, \tag{25}$$

$$m = \frac{2(\delta + g - \eta_A)}{\sigma^2},$$

$$q = 2\,(1 - \frac{\delta - \eta_L}{\sigma^2}),$$

$$\alpha = \frac{2\,(\eta_L - \delta) + 3\sigma^2 + \sqrt{8\lambda\sigma^2 + (2\eta_L - 2\delta - \sigma^2)^2}}{2\,\sigma^2},$$

$$\gamma = 1 + \frac{\sqrt{8\lambda\sigma^2 + (2\eta_L - 2\delta - \sigma^2)^2}}{\sigma^2},$$

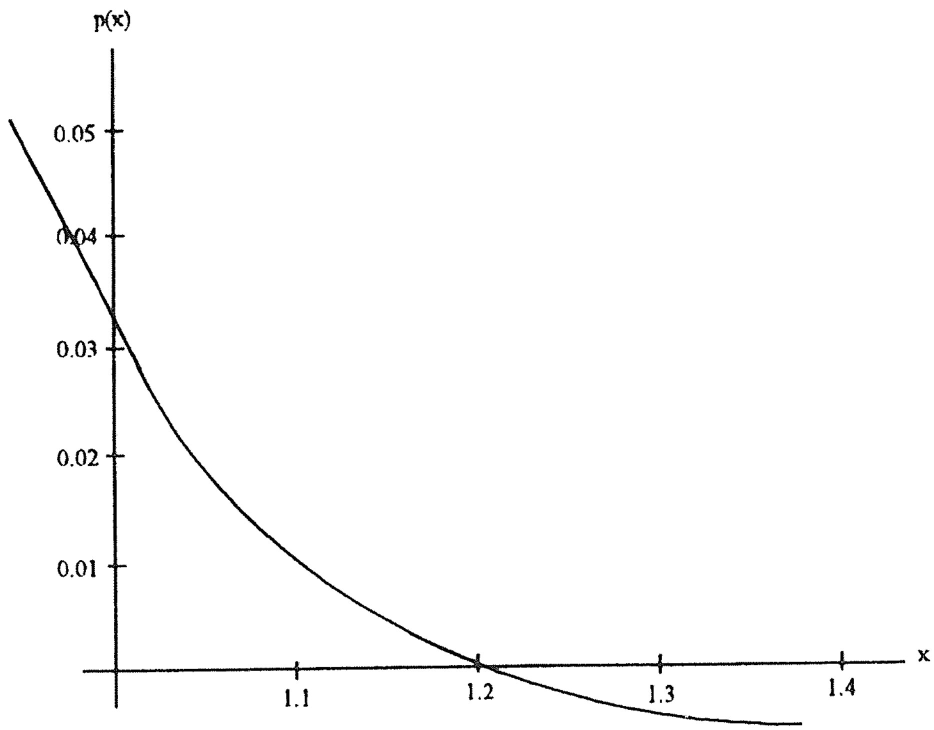
and a_1, a_2, and b_1 are the constants of integration, whose values are determined by the boundary conditions. Figure 1 shows the form of the solution for some specific parameter values.

At the start of the insurance contract the PBGC charges the rate g that makes the insurance pension net of premium payments equal to zero. This value of g is the rate payment equivalent to the present value of the pension insurance, i.e., if the PBGC charges this rate it would break even in expected value. We denote this actuarially fair premium rate by g_a. In Section II we estimate g_a for a sample of US firms with the method of Newton for finding roots.

Figure 1

The Pension Put per Unit Liability Function

The figure was computed for the Model 1 of pension insurance, and with the following parameter values: $c = 0.005$, $\sigma = 0.2$, $\eta_A = 0.07$, $\eta_L = 0.0$, $\delta = 0.11$ and $g = 0.015$. As the figure illustrates, in this example the actuarially fair premium rate for a pension asset to pension liability ratio of 1.2 is 1.5% of the pension liabilities.



B. TIME INVARIANT PREMIUM RATES

In this model the PBGC enters in a contractual agreement with a firm to insure its defined benefit pensions at a constant premium rate g. The PBGC conducts random audits that follow a Poisson process with characteristic parameter λ. Such audits determine the financial situation of the pension fund. If the fund is found solvent, the pension plan is allowed to continue operating, and the insurance is provided at the same premium rate g. If the fund is found insolvent, the pension plan is terminated, with the PBGC absorbing the difference between pension liabilities and pension assets.[14] A new pension fund will be started for the insured workers, this time, of the defined contribution type. Again we are interested in obtaining a fair price for the insurance provided by the PBGC. If we let $P = P(A,L)$ represent the value of the premium insurance we can argue, as before, that since the time between audits is exponentially distributed with characteristic parameter λ, $P(.)$ does not depend on the time to maturity, nor does it depend on the time since last audit (see footnote 9).

By Ito's Lema, (5) holds again. However, in case 1 (pension plan solvent) the plan is not terminated after an audit, so we now have:

Case 1. Audit and pension plan solvent ($A \geq L$),

$$\Delta P = C(L). \tag{26}$$

This implies that the expected change of the value of the pension put per unit time in the solvent case is

$$P \alpha_P = \phi \, [P(A,L)] + \lambda \, C(L), \tag{27}$$

which modifies the differential equation in case that the plan be found solvent as

$$\begin{gathered}\frac{1}{2} [A^2 \sigma_A^2 \frac{\partial^2 P}{\partial A^2} + L^2 \sigma_L^2 \frac{\partial^2 P}{\partial L^2} + 2 A L \sigma_A \sigma_L \rho \frac{\partial^2 P}{\partial A \, \partial L}] \\ + [\eta_A A - \delta L - g L + A r] \frac{\partial P}{\partial A} + [\eta_L - \delta + r] L \frac{\partial P}{\partial L} \\ - r P + \lambda c L - g L = 0.\end{gathered} \tag{28}$$

Case 2. Audit and pension plan insolvent ($A < L$). In this case (8), (10) and (15) still hold.

[14]Under ERISA a pension plan may be terminated voluntarily by the firm or involuntarily by the PBGC, upon court order, if the plan 1) fails to meet the minimum funding standards, 2) is unable to pay benefits when due, 3) is administered improperly, or 4) if the liability of the PBGC for fulfilling claims deriving from the plan is likely to increase unreasonably. In our stylized model, reasons 1) and 2) above motivate the audits. In the event of case 2), we assume that an audit will be conducted with probability one. Since the expected time between audits is $1/\lambda$, this is formally equivalent to assuming that if the plan fails to pay its retirees, λ jumps to infinity. The same would occur if the firm fails to pay the insurance premium.

After substituting $p = P/L$, $x = A/L$ and defining σ^2 as in (16), we obtain ordinary differential equations. Only the one corresponding to the interval $x \geq 1$ is different, but we state both for completeness:

Case 1. Pension plan solvent ($x \geq 1$):

$$\frac{\sigma^2}{2} x^2 \frac{d^2p}{dx^2} + [(\delta - \eta_L) x - (\delta + g - \eta_A)] \frac{dp}{dx} - (\delta - \eta_L) p - g + \lambda c = 0. \tag{29}$$

Case 2. Pension plan insolvent ($0 \leq x < 1$); same as (18):

$$\frac{\sigma^2}{2} x^2 \frac{d^2p}{dx^2} + [(\delta - \eta_L) x - (\delta + g - \eta_A)] \frac{dp}{dx} - (\delta + \lambda - \eta_L) p - g + \lambda (c + 1 - x) = 0. \tag{30}$$

The solutions are subject to the following boundary conditions:[15]

1. Continuity of the pension put function,

$$p_-(1) = p_+(1). \tag{31}$$

2. Continuity of the first derivative,

$$\frac{dp_-}{dx}(1) = \frac{dp_+}{dx}(1). \tag{32}$$

3. Value of p at zero,

$$p_-(0) = 1 + c. \tag{33}$$

4. Value of p at infinity,

$$p_+(x) \text{ is bounded as } x \to \infty. \tag{34}$$

[15]The intuition for the boundary conditions is the same as for model 1.

The solution to these differential equations is

$$p_{-}(x) = -x + \frac{\delta - \eta_A + \lambda(c+1)}{\lambda + \delta - \eta_L} \tag{35}$$

$$+ a_1 e^{-\frac{m}{x}} x^{-\frac{\gamma - q}{2}} M(\alpha, \gamma, \frac{m}{x}) + \overline{a_2}\, m^{1-\gamma} e^{-\frac{m}{x}} X^{\frac{\gamma - l}{2}} M(\alpha - \gamma + 1, 2 - \gamma, \frac{m}{x})$$

and

$$p_{.}(x) = \frac{\lambda c - g}{\delta - \eta_L} + e^{-\frac{m}{x}} x^{-l}\, \overline{b_1}\, M(2, 2+l, \frac{m}{x}), \tag{36}$$

where the new constants of integration are represented by a bar over them. Note that a_1 is the same as in model 1. As before, the actuarially fair insurance premium rates can be computed by numerical methods.

II. Empirical Estimates

In this section we present estimates for actuarially fair PBGC insurance premiums corresponding to the two pension insurance models developed in the previous section.

A. Data

Starting in 1983, *Pensions and Insurance Age* has published an annual article containing pension fund statistics derived from the previous year reports of the Fortune 100 companies ranked by sales. The survey includes total pension benefits, defined as the actuarial present value of benefits promised to all past and present employees, based on service already performed, the assumed discount rate used in its computation, the market value of the pension fund assets and the pension expense, i.e., the amount charged against income to fund benefit payments in a given year.

Unfortunately, from 1988 on *Pensions and Investment Age* has not published the assumed discount rate which was used in computing the present value of the pension benefits. Also, in 1991 they restricted their attention to the

Fortune 50 companies. For this reason, we decided to include in this study only those companies that were ranked in the Fortune 100 in every year from 1982 to 1986. After accounting for missing information on some of them, the sample was reduced to 58 companies. Following is a description of the way in which the variables and parameters of the model were estimated.

1) Pension fund assets (A). In order to obtain the value of the assets backing the pension fund, we added 30 per cent of the value of equity of the companies at year end (obtained from the CRISP data set) to the market value of pension assets reported in the survey.

2) Pension fund liabilities (L). An obvious problem with the pension benefit figures reported in the survey is that they critically depend on the discount rate assumed in the computation of their present value, which varies considerably across firms (from 7% to 14.8%) and also over time. To obtain a better measure we substituted for this reported rate of return[16] with the long-term market interest rate for each year. Since we do not have information on the time paths of the pension payments of each plan, we assumed that the cash flows are constant over time. This implies that the present value of pension payments is that of a perpetuity, and therefore we can approximate the market value of pension fund liabilities by multiplying the reported pension benefits by the ratio of the assumed discount rate of the plan to the average rate on a 30-year US treasury notes and bonds.[17]

3) Rate of contributions by the firm to its pension fund (η_A). The survey reports the pension expense for each year, which varies over time and was even negative for a few firms in the last two years of the sample. Since our model assumes that this rate is constant, we approximate this parameter by the ratio of the average pension expense per year to the average market value of pension liabilities per year. The intuition behind this procedure is that firms may change their pension expense from year to year, but that they have a target average contribution.

4) Rate of increase in pension liabilities due to new workers (η_L). Since we were dealing with big companies that are well established, we assumed that

[16]In a few instances there are two discount rates reported for a firm in a given year, presumably corresponding to different pension plans; when this was the case we took the arithmetic mean as the assumed discount rate.

[17]The average annual rates for 30-year US Treasury notes and bonds in the period 1982-86 were 12.76%, 10.84%, 12.41%, 10.71% and 7.78%, respectively. Source: *Annual Statistical Digest 1980-1989*, Federal Reserve System, Washington, D.C.

the number of employees is stationary over the sample period, which implies that this parameter will be zero.

5) Rate of payments to retirees (δ). The assumption that the pension benefits are a perpetuity made to approximate the market value of pension liabilities suggests an approximation for the rate of payments to retirees. This rate is defined as the ratio of pension payments to pension liabilities. But we saw that an approximation to the market value of the pension liabilities is the ratio of pension payments to the long-term interest rate. Therefore, we use the average long-term interest rate as an approximation for δ.[18]

6) Volatility (σ). We obtained an estimate for the volatility using the pension assets and pension liabilities data. Admittedly this estimate is not accurate, since our sample is small. It would have been possible to improve the estimates of the variance of the asset side, since there is a lot more information on the value of the equity and we also could have made sensible assumptions about the portfolio composition of the assets of the pension fund. But it is not possible to obtain more information on the pension liabilities or on the covariance between pension assets and pension liabilities.

7) Auditing costs (c). We assume that the auditing costs would be of the order of 0.5% of pension liabilities, and do sensitivity analysis considering also values of 1% and 0.1%.

8) Periodicity of the audits (λ). Of course this parameter was not estimated, since the system in practice is not one of random audits following a Poisson process. Because the current legislation requires firms to report to the PBGC once a year, we set $\lambda = 1.0$, implying one audit per year on average. In Section III we treat the frequency of the audits as an endogenous variable.

As it can be shown, the equilibrium contribution rate to the pension plan, η_A, satisfies the inequality $\eta_A \leq \delta + g$. The intuition for this restriction comes from the assumption that the parameters of the model are constant over time: the constant optimal contribution rate to the pension plan is such that as time increases the pension plan surplus converges to zero. If the firm contributes a rate η_A superior to the rate of payment to retirees δ and the rate of payment to the PBGC g, the pension plan would converge to a positive surplus as times goes on. Of course, in reality the contribution rates vary over time, and for this reason some of the firms in our sample do not satisfy the steady-state restriction. We decided to take them out of the sample, which leaves us with 38 firms.

[18]The average long term rate of return was 10.92% in the period 1982-86.

Table 3 reports the parameter estimates for the remaining firms. The pension asset to pension liability ratio varies greatly among firms, from 0.57 for Bethlehem Steel up to 11.7 for Phillips Petroleum. Of course, the amount of overfunding is not as big as these numbers would suggest. Recall that "assets" are the assets backing the pension plan, which include 30% of the value of equity, which is very big for the firms that appear with high overfunding. For example, in the case of Phillips Petroleum, 30% of the Equity accounted for 84% of the assets backing the pension plan, for 91% in the case of Occidental Petroleum, and for 77% in the cases of Motorola and Amerada Hess.

Table 3

Parameter Estimates for 38 Fortune 100 Companies

FIRM	ASSET - LIABILITY RATIO (x)	VOLATILITY (σ)	RATE OF CONTRIBUTION (η_A)
Amerada Hess	7.656	.227	.042
Ashland Oil	3.303	.149	.042
Atlantic Richfield	3.008	.277	.083
Bethlehem Steel	.570	.144	.075
Boeing	1.992	.303	.096
Borden	2.745	.572	.038
Caterpillar	1.583	.127	.063
Chrysler	.781	.328	.065
Coastal	2.041	.650	.052
Colgate-Palmolive	2.726	.332	.077
E.I. Du Pont	2.021	.202	.036
Eastman Kodak	2.686	.149	.056
Ford Motor	1.390	.137	.077
General Dynamics	1.608	.352	.078
General Electric	2.448	.363	.061
General Motors	1.154	.200	.069
Goodyear Tire & Rubber	2.232	.086	.062

(Continues)

Table 3 *(Continued)*

Parameter Estimates for 38 Fortune 100 Companies

FIRM	ASSET-LIABILITY RATIO (x)	VOLATILITY (σ)	RATE OF CONTRIBUTION (η_A)
W. R. Grace	3.803	.374	.098
Honeywell	1.286	.501	.074
I.C. Industries	2.619	.280	.079
International Paper	2.237	.134	.010
Lockheed	1.636	.274	.059
L T V	1.177	.155	.080
Mc Donnell Douglas	1.909	.270	.043
3 M	4.482	.164	.108
Mobil	2.857	.187	.110
Monsanto	1.877	.200	.068
Motorola	8.327	.512	.062
Occidental Petroleum	9.944	.325	.046
Philip Morris	3.755	.961	92
Phillips Petroleum	16.781	.451	.012
Rockwell International	1.738	.173	.073
Standard Oil (Ohio)	3.947	.255	.095
T R W	1.550	.372	.098
Union Carbide	1.459	.279	.096
United Technology	1.451	.421	.076
Westinghouse Electric	1.250	.375	.037
Weyerhaeuser	3.296	.073	.081

The estimates for volatility corresponding to two firms in the sample are unreasonable large: Coastal (σ = 0.65) and Philip Morris (σ = 0.961). In both cases the high volatility can be attributable to a special event occurring between 1984 and 1985: in the case of Coastal, the reported pension assets increased by 141% (from 154.4 mill to 571.5 mill) while the reported pension liabilities increased by 166% (from 91.4 mill to 242.5 mill), with no change in the reported discount rate (of 8% in both years). Similarly, in the case of Philip Morris the reported pension assets increased by 217% (from 745.1 mill to 2,361 mill) and the reported pension liabilities in 207% (from 537.7 mill to 1,654 mill), again with no change in the reported discount rate (of 7.5% both years). Unfortunately, the information available is very aggregated, so we do not know what happened. The magnitude of the changes, however, suggests that they incorporated in the report of 1985 other pension plans, not previously included. Since the purpose of this section is to illustrate the premium rates that this model suggests, which are critically affected by the volatility, we exclude these two firms in the estimation of the premium rates reported in Table 4.

B. RESULTS

The premium rates for the 38 companies of the sample for the year 1986 are presented in Table 4. These actuarially fair premium rates are defined as the premium rates g which make the function $p_+(x)$ zero, i.e., the roots of the pension put per unit liability defined by (24) for model 1 and by (36) for model 2. They were computed using the Mathematica software (see Wolfan 1991), which uses Newton's method to numerically obtain roots when an analytic solution does not exist.[19]

Two companies in the sample had a pension asset to pension liabilities ratio less than 1.0 (Bethlehem Steel and Chrysler), so that according to the assumptions of our model their pension plans should have been terminated by the PBGC and new pension plans instituted for their employees, this time of the defined contribution type.

As it should be expected, in the case of very safe pension plans (those characterized by big asset to liabilities ratio and/or small volatility) the auditing

[19]In order to find a solution to the equation $f(x) = 0$, the method starts with an initial value x_0 and uses the derivative of the function $f(x)$, $f'(x)$, to take a sequence of steps toward a solution, according to the formula:

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}.$$

(See p. 694 in Wolfram 1991).

costs constitute most of the insurance premiums, since really there is no risk involved. This would be the case of Amerada Hess, for example, with an asset to liability ratio of 7.656 and volatility of 0.227. Obviously, in these cases the assumption about the auditing costs is critical to compute the premiums rates. As the funding decreases, however, the option component in the risk premium increases considerably. For example, in the case of General Motors, with an asset to liability of 1.154 and volatility of 0.2 the first model, with audit costs of 0.5% of liabilities, gives a premium rate of 2% of liabilities per year, which is much bigger than the one corresponding to Amerada Hess (0.5% of liabilities per year), and the assumption relative to auditing costs, though important, is not as critical.

Table 4

Insurance Premiums Estimates for 1986

FIRM	$g1$ c=.01	$g2$ c=.01	$g1$ c=.005	$g2$ c=.005	$g1$ c=.001	$g2$ c=.001
Amerada Hess	.01000	.01000	.00500	.00500	.00100	.00100
Ashland Oil	.01000	.01000	.00500	.00500	.00100	.00100
Atlantic Richfield	.01008	.01065	.00507	.00560	.00107	.00156
Bethlehem Steel	---	---	---	---	---	---
Boeing	.01170	.01282	.00661	.00766	.00255	.00353
Borden	.02969	.03902	.02427	.03340	.01994	.02891
Caterpillar	.01001	.01005	.00501	.00504	.00101	.00103
Chrysler	---	---	---	---	---	---
Colgate-Palmolive	.01075	.01263	.00571	.00750	.00169	.00339
E.I. Du Pont	.01018	.01083	.00516	.00574	.00115	.00168
Eastman Kodak	.01000	.01000	.00500	.00500	.00100	.00100
Ford Motor	.01018	.01020	.00515	.00517	.00113	.00115
General Dynamics	.02317	.02113	.01768	.01563	.01330	.01124
General Electric	.01260	.01579	.00753	.01054	.00342	.00635
General Motors	.02635	.01642	.02032	.01088	.01554	.00648

(Continues)

Table 4 *(Continued)*

Insurance Premiums Estimates for 1986

FIRM	$g1$ c=.01	$g2$ c=.01	$g1$ c=.005	$g2$ c=.005	$g1$ c=.001	$g2$ c=.001
Goodyear Tire & Rubber	.01000	.01000	.00500	.00500	.00100	.00100
W. R. Grace	.01035	.01218	.00533	.00708	.00132	.00301
Honeywell	.11903	.07047	.11115	.06352	.10492	.05800
I.C. Industries	.01023	.01109	.00521	.00601	.00120	.00195
International Paper	.01000	.01006	.00500	.00504	.00100	.00103
Lockheed	.01457	.01509	.00933	.00977	.00515	.00553
L T V	.01401	.01166	.00865	.00647	.00439	.00234
Mc Donnell Douglas	.01195	.01367	.00684	.00844	.00275	.00425
3 M	.01000	.01000	.00500	.00500	.00100	.00100
Mobil	.01000	.01007	.00500	.00500	.00100	.00100
Monsanto	.01017	.01052	.00515	.00546	.00114	.00142
Motorola	.01046	.01526	.00545	.01014	.00144	.00604
Occidental Petroleum	.01000	.01023	.00500	.00521	.00100	.00120
Phillips Petroleum	.01001	.01162	.00501	.00658	.00101	.00254
Rockwell International	.01009	.01025	.00507	.00521	.00107	.00119
Standard Oil (Ohio)	.01000	.01011	.00500	.00510	.00100	.00109
T R W	.02692	.02247	.02133	.01695	.01688	.01254
Union Carbide	.01699	.01500	.01165	.00969	.00738	.00546
United Technology	.05044	.03645	.04424	.03048	.03930	.02573
Westinghouse Electric	.08528	.05094	.07773	.04410	.07174	.03870
Weyerhaeuser	.01000	.01000	.00500	.00500	.00100	.00100

$g1$ = Actuarially fair premium rates for model 1

$g2$ = Actuarially fair premium rates for model 2

c = Audit cost per unit of pension liabilities

--- = Means that the pension plan should be terminated, since pension assets are insufficient to cover pension liabilities.

An interesting question concerns which insurance model gives higher premium rates. Since the boundary conditions are the same for the two models, the differences in premium rates must be found in two factors: the risk-adjusted drift and the discount rate. The risk-adjusted drift is the same for both equations (see (17) and (29)), and is given by the coefficient of *dp/dx*:

$$(\delta - \eta_L)\, x - (\delta + g - \eta_A). \tag{37}$$

If this coefficient is positive, it means that the pension asset to pension liability ratio will probably increase in the future. In this case the next premium rates will probably be lower in a contract where the premium rates are revised after audits (model 1). In contrast, in the case of constant premium rates (model 2), the terms of the contract cannot be altered and therefore the current premium rates charged must be lower, to reflect this future improvement in the risk of the pension plan.

The parameter that plays the role of a discount rate is the coefficient of p and is different in each case; for the model of premium rates revised after each audit it is $(\lambda + \delta - \eta_L)$, and for the model of time-invariant premium rates it is $(\delta - \eta_L)$. This means that the discount rate is higher for the first model, implying a lower present value of pension payments and therefore lower premium rates. This effect is very strong in our estimates, since we used $\lambda = 1.0$, $\eta_L = 0.0$, and $\delta = 0.1092$. For this reason, in 75% of the firms in our sample the premium rates corresponding to the first model (premium rates revised after each audit) were smaller.

Table 4 suggests three comments on the current pricing schedule. First of all, there seems to be a lot more differences in the actuarially fair premium rates than is permitted under current law.[20] For example, looking at the column corresponding to model 1 and an audit cost of 0.1%, we see that the premium rates range from 0.1% of pension liabilities in the case of "safe plans", like Ashland Oil (with pension asset to pension liability ratio of 3.3 and a volatility of 0.149) to 10% of pension liabilities for a "risky plan", like Honeywell

[20]As we mentioned in the introduction, the pension insurance premiums are currently $19 per participant plus $9 per $1,000 of unfunded vested benefits, capped at $53 per participant for a total maximum premium of $72 per participant. Note that we should expect that our model give higher insurance premiums than those actually charged, since under the current system there is a maximum guaranteed pension ($2,250.00 per month in 1991), while in our model pension benefits are fully insured. But the point is that the variation in premium rates is much higher than what could reasonably be attributed to this factor.

(characterized by a ratio of pension assets to pension liabilities of 1.2 and a volatility of 0.501).

A second observation is that the option nature of the problem implies actuarially fair pension premiums that grow at an increasing rate as the pension asset to pension liability ratio decreases, in sharp contrast with the current pricing scheme of flat premiums after a certain level of underfunding (see the convex form of the *p(x)* function in figure 1). This suggests that the largest subsidies implied by the current pricing policy of the PBGC are in favor of firms whose pension plans are more underfunded.

The third comment is that under the current pricing no consideration is given to other indicators of risk besides the amount of underfunding. Thus, measures of volatility of the pension assets and liabilities, the firm's contribution to its pension plan, or the rate of payments to retirees are not considered. To the extent that these parameters differ across firms in the economy -and the data that we have suggest so (see Table 3)- there are cross subsidies in the economy in favor of the more risky plans and in detriment of the PBGC and possibly the safer plans.

III. Optimal Auditing

In this section we interpret the parameter λ that characterizes the Poisson process as a policy variable instead of as an exogenously specified constant, and we ask what frequency of audits λ is optimal from a social perspective.

From the perspective of both the insured workers and the PBGC the frequency of the audits is a matter of indifference: the workers are fully insured, no matter how frequently the audits are conducted, and for any audit frequency the PBGC will charge actuarially fair premium rates, breaking even on average.

This implies that from a social point of view the optimal auditing policy is the one that minimizes the cost of the insurance to the firms. In other words, the optimal amount of resources allocated to the auditing process is that which minimizes the value of the pension put.

Monitoring constitutes an important incentive for firms to be more careful in their funding policies. In order to incorporate this observation in our model in a parsimonious way we assume that the rate of contributions η_A is proportional to the frequency of the audits as measured by λ:

$$\eta_A = k\ \lambda. \tag{38}$$

Thus, more auditing represented by a higher λ would ensure more funding by increasing the rate of contributions η_A.

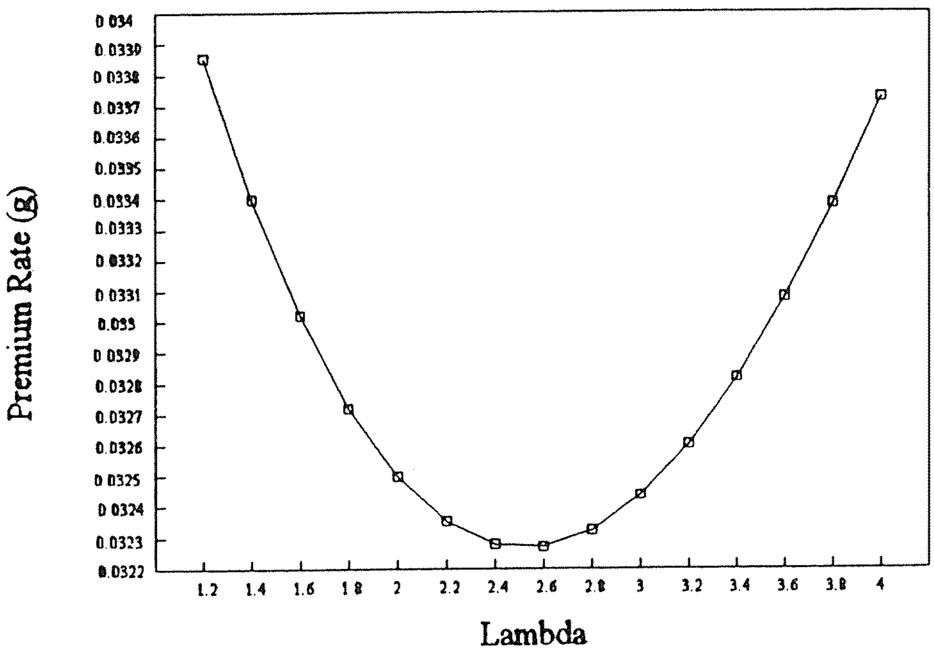
As we have seen, in the structure of our model the periodicity of the audits is determined (in expected value) by the parameter λ that characterizes the Poisson process. The argument is that the government should choose λ so as to minimize the actuarially fair premium rate g_a:

$$\lambda^* = \textbf{Argmin} \quad [g_a\ (\lambda)]. \tag{39}$$

The economics of the problem suggests the existence and uniqueness of an interior solution: decreasing λ increases the expected time between audits (which is $1/\lambda$) and so results in less monitoring on average. Less monitoring reduces the surveillance costs, but it also increases the risk of the PBGC having to cover a large amount of underfunding, and this for two reasons: less monitoring induces the firm to contribute less to its pension plan and, since more time elapses between audits, there is a bigger chance for the asset to liability ratio to fall dangerously below 1.0. From the balance of this trade-off emerges an optimal auditing frequency λ^*. As an illustration, we computed the actuarially fair premium rates corresponding to different values of λ for the case of General Motors in 1986, using the first model of pension insurance, and assuming that the proportionality factor $k = 0.02$, and that the auditing costs are $c = 0.005$. The results are plotted in figure 2; the U-shape of the relationship reflects the discussed trade-off; in this example the optimal monitoring frequency is 2.5 audits per year on average.

Figure 2

Optimal Monitoring Frequency



The optimal auditing frequency is dependent on the parameters of the model. Table 5 reports a summary of the comparative statics results, whose intuition is straightforward:

1) Higher audit costs induce less monitoring and increase the premium rates.

2) Higher volatility induces more monitoring and implies more expensive insurance both because of higher risk per unit time and higher monitoring costs.

3) Larger rates of increase in insured workers imply that the asset to liability ratio will tend to be lower (it lowers the risk adjusted drift) and the risk higher, requiring more monitoring and increasing the insurance costs. Technically speaking, a higher η_L also decreases the discount rate, increasing the present value of pension insurance, thus reinforcing the effect on higher premium rates.

4) Higher pension asset to pension liability ratios will decrease the need for monitoring and lower the premium rates. Since the risk-adjusted drift increases, it makes it safer in the future too.

5) A higher rate of payments to retirees δ increases the risk-adjusted drift, making it likely for the pension asset to pension liability ration to increase in the future (provided that the pension plan is solvent). This implies that less auditing will be required and that the premium rates can be lowered, both because of less risk and less auditing costs. Moreover, a higher δ also increases the discount rate, lowering the present value of pension insurance, thus reinforcing the reduction in premium rates.

6) A higher semi-elasticity of contributions k induces two effects on optimal auditing frequency: it tends to increase the optimal frequency since the auditing is more "productive" in that it has a bigger impact on the contributions of the firm, but it also tends to reduce it, since the firm is contributing more to the pension plan, making auditing less necessary. However, the effect on premium rates is unambiguous: for a given audit frequency λ, more contributions to the pension plan make it safer (the risk-adjusted drift is reduced), and therefore the required premium rates are lower. Now it is true that higher k may induce more auditing, but this will lower the premium rates even more, since we are dealing with optimal auditing.

Table 5

Comparative Statics of the Optimal Auditing

VARIABLE	OPTIMAL PERIODICITY OF AUDITS (λ^*)	OPTIMAL ACTUARIALLY FAIR PREMIUM RATE (g_a^*)
Auditing costs (c)	Decrease (-)	Increase (+)
Volatility (σ)	Increase (+)	Increase (+)
Rate of demographical increase in workers (η_L)	Increase (+)	Increase (+)
Pension asset to liability ratio (x)	Decrease (-)	Decrease (-)
Rate of payments to retired workers (δ)	Decrease (-)	Decrease (-)
Semi-elasticity of contributions to the pension plan (k)	Parameter dependent (?)	Decrease (-)

IV. Conclusions

We have developed two models of pension insurance, one where the premium rates are revised after each audit and one where they are fixed until the end of the insurance contract. We have used data from a sample of US firms to estimate the parameters of the model, and computed actuarially fair risk-based premium rates. We also discussed optimality of the frequency of the audits.

The main implication of the model for the pension insurance provided by the government under ERISA is that the premium rates charged on grossly underfunded pension plans are clearly inadequate. As we have seen, the actuarially fair premium rates grow at an increasing rate when the pension assets to pension liabilities ratio decreases. The actual pricing policy, instead, is to charge a flat rate after certain underfunding. A second implication is that risk-based premium rates depend critically on other variables, like the volatility of the assets and liabilities of the pension plan.

An interesting extension of this model would be to allow the parameters of the model to vary over time. For example, a new firm will be hiring more workers in an initial period ($\eta_L > 0$); then, after it reaches maturity, it may keep a stable number of employees ($\eta_L = 0$), and perhaps over a period of intense competition and decline it may reduce its size ($\eta_L < 0$). This extension would make the model usable for a wider set of firms.

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